

8.5

$$1) \frac{4+2\sqrt{3}}{\sqrt{9}} = \frac{4+2\sqrt{3}}{3}$$

$$3) \frac{4+2\sqrt{3}}{5\sqrt{4}} = \frac{4+2\sqrt{3}}{5 \cdot 2} = \frac{4+2\sqrt{3}}{10} = \frac{2(2+\sqrt{3})}{10} = \frac{2+\sqrt{3}}{5}$$

$$5) \frac{2-5\sqrt{5}}{4\sqrt{13}} \left(\frac{\sqrt{13}}{\sqrt{13}} \right) = \frac{2\sqrt{13}-5\sqrt{65}}{4 \cdot 13} = \frac{2\sqrt{13}-5\sqrt{65}}{52}$$

$$7) \frac{\sqrt{2}-3\sqrt{3}}{\sqrt{3}} \left(\frac{\sqrt{3}}{\sqrt{3}} \right) = \frac{\sqrt{6}-3\sqrt{9}}{3} = \frac{\sqrt{6}-3 \cdot 3}{3} = \frac{\sqrt{6}-9}{3}$$

$$9) \frac{2p+3\sqrt{5p^4}}{5\sqrt{20p^2}} = \frac{2p+3\sqrt{5p^4}}{5\sqrt{2^2 \cdot 5p^2}} = \frac{2p+3p^2\sqrt{5}}{5 \cdot 2p\sqrt{5}} = \frac{2p+3p^2\sqrt{5}}{10p\sqrt{5}} \left(\frac{\sqrt{5}}{\sqrt{5}} \right) = \frac{2p\sqrt{5}+3p^2\sqrt{25}}{10p(5)} = \frac{2p\sqrt{5}+3p^2 \cdot 5}{50p} =$$

$$\frac{2p\sqrt{5}+15p^2}{50p} = \frac{p(2\sqrt{5}+15p)}{50p} = \frac{2\sqrt{5}+15p}{50}$$

$$11) \frac{\sqrt{3m^2-4\sqrt{2m^4}}}{5\sqrt{12m^4}} = \frac{\sqrt{3m^2-4\sqrt{2m^4}}}{5\sqrt{2^2 \cdot 3m^4}} = \frac{m\sqrt{3-4m^2\sqrt{2}}}{2 \cdot 5m^2\sqrt{3}} = \frac{m\sqrt{3-4m^2\sqrt{2}}}{10m^2\sqrt{3}} \left(\frac{\sqrt{3}}{\sqrt{3}} \right) = \frac{m\sqrt{9-4m^2\sqrt{6}}}{10m^2 \cdot 3} =$$

$$\frac{3m-4m^2\sqrt{6}}{30m^2} = \frac{m(3-4m\sqrt{6})}{30m^2} = \frac{3-4m\sqrt{6}}{30m}$$

$$13) \frac{5}{3\sqrt{5}+\sqrt{2}} \left(\frac{3\sqrt{5}+2}{3\sqrt{5}+2} \right) = \frac{15\sqrt{5}-5\sqrt{2}}{9 \cdot 5-2} = \frac{15\sqrt{5}-5\sqrt{2}}{45-2} = \frac{15\sqrt{5}-5\sqrt{2}}{43}$$

$$15) \frac{2}{5+\sqrt{2}} \left(\frac{5-\sqrt{2}}{5-\sqrt{2}} \right) = \frac{10-2\sqrt{2}}{25-2} = \frac{10-2\sqrt{2}}{23}$$

$$17) \frac{3}{4-3\sqrt{3}} \left(\frac{4+3\sqrt{3}}{4+3\sqrt{3}} \right) = \frac{12+9\sqrt{3}}{16-9 \cdot 3} = \frac{12+9\sqrt{3}}{16-27} = \frac{12+9\sqrt{3}}{-11}$$

$$19) \frac{4}{3+\sqrt{5}} \left(\frac{3-\sqrt{5}}{3-\sqrt{5}} \right) = \frac{12-4\sqrt{5}}{9-5} = \frac{12-4\sqrt{5}}{4} = \frac{4(3-\sqrt{5})}{4} = 3 - \sqrt{5}$$

$$21) \frac{-4}{4-4\sqrt{2}} \left(\frac{4+4\sqrt{2}}{4+4\sqrt{2}} \right) = \frac{-16-16\sqrt{2}}{16-16 \cdot 2} = \frac{-16-16\sqrt{2}}{16-32} = \frac{-16-16\sqrt{2}}{-16} = \frac{-16(1+\sqrt{2})}{-16} = 1 + \sqrt{2}$$

$$23) \frac{5}{\sqrt{n^4-5}} = \frac{5}{n^2-5}$$

$$25) \frac{4p}{3-5\sqrt{p^4}} = \frac{4p}{3-5p^2}$$

$$27) \frac{4}{5+\sqrt{5x^2}} = \frac{4}{5+x\sqrt{5}} \left(\frac{5-x\sqrt{5}}{5-x\sqrt{5}} \right) = \frac{20-4x\sqrt{5}}{25+5x^2}$$

$$29) \frac{5}{2+\sqrt{5r^3}} = \frac{5}{2+r\sqrt{5r}} \left(\frac{2-r\sqrt{5r}}{2-r\sqrt{5r}} \right) = \frac{10-5r\sqrt{5r}}{4-r^2(5r)} = \frac{10-5r\sqrt{5r}}{4-5r^3}$$

$$31) \frac{5}{-5v-3\sqrt{v}} \left(\frac{-5v+3\sqrt{v}}{-5v+3\sqrt{v}} \right) = \frac{-25v+15\sqrt{v}}{25v^2-9v}$$

$$33) \frac{4\sqrt{2}+3}{3\sqrt{2}+\sqrt{3}} \left(\frac{3\sqrt{2}-\sqrt{3}}{3\sqrt{2}-\sqrt{3}} \right) = \frac{12\sqrt{4}-4\sqrt{6}+9\sqrt{2}-3\sqrt{3}}{9 \cdot 2 - 3} = \frac{12 \cdot 2 - 4\sqrt{6} + 9\sqrt{2} - 3\sqrt{3}}{18-3} = \frac{24-4\sqrt{6}+9\sqrt{2}-3\sqrt{3}}{15}$$

$$35) \frac{2-\sqrt{5}}{-3+\sqrt{5}} \left(\frac{-3-\sqrt{5}}{-3-\sqrt{5}} \right) = \frac{-6-2\sqrt{5}+3\sqrt{5}+\sqrt{25}}{9-5} = \frac{-6-2\sqrt{5}+3\sqrt{5}+5}{4} = \frac{-1+\sqrt{5}}{4}$$

$$37) \frac{5\sqrt{2}+\sqrt{3}}{5+5\sqrt{2}} \left(\frac{5-5\sqrt{2}}{5-5\sqrt{2}} \right) = \frac{25\sqrt{2}-25\sqrt{4}+5\sqrt{3}-5\sqrt{6}}{25-25 \cdot 2} = \frac{25\sqrt{2}-25 \cdot 2+5\sqrt{3}-5\sqrt{6}}{25-50} =$$

$$\frac{25\sqrt{2}-50+5\sqrt{3}-5\sqrt{6}}{-25} = \frac{5(5\sqrt{2}-10+\sqrt{3}-\sqrt{6})}{-25} = \frac{5\sqrt{2}-10+\sqrt{3}-\sqrt{6}}{-5}$$

$$39) \frac{\sqrt{3}+\sqrt{2}}{2\sqrt{3}-\sqrt{2}} \left(\frac{2\sqrt{3}+\sqrt{2}}{2\sqrt{3}+\sqrt{2}} \right) = \frac{2\sqrt{9}+\sqrt{6}+2\sqrt{6}+\sqrt{4}}{4 \cdot 3 - 2} = \frac{2 \cdot 3 + \sqrt{6} + 2\sqrt{6} + 2}{12-2} = \frac{6+\sqrt{6}+2\sqrt{6}+2}{10} = \frac{8+3\sqrt{6}}{10}$$

$$41) \frac{\sqrt{3}-\sqrt{2}}{4+\sqrt{5}} \left(\frac{4-\sqrt{5}}{4-\sqrt{5}} \right) = \frac{4\sqrt{3}-\sqrt{15}-4\sqrt{2}+\sqrt{10}}{16-5} = \frac{4\sqrt{3}-\sqrt{15}-4\sqrt{2}+\sqrt{10}}{11}$$

$$43) \frac{4+2\sqrt{2x^2}}{5+2\sqrt{5x^3}} = \frac{4+2x\sqrt{2}}{5+2x\sqrt{5x}} \left(\frac{5-2x\sqrt{5x}}{5-2x\sqrt{5x}} \right) = \frac{20+8x\sqrt{5x}+10x\sqrt{2}-4x^2\sqrt{10x}}{25-4x^2(5x)} =$$

$$\frac{20+8x\sqrt{5x}+10x\sqrt{2}-4x^2\sqrt{10x}}{25-20x^3}$$

$$45) \frac{(2\sqrt{3m^2}-\sqrt{2m^4})}{5-\sqrt{3m^2}} = \frac{(2m\sqrt{3}-m^2\sqrt{2})}{5-m\sqrt{3}} \left(\frac{5+m\sqrt{3}}{5+m\sqrt{3}} \right) = \frac{10m\sqrt{3}+2m^2\sqrt{9}-5m^2\sqrt{2}-m^3\sqrt{6}}{25-3m^2} =$$

$$\frac{10m\sqrt{3}+2m^2(3)-5m^2\sqrt{2}-m^3\sqrt{6}}{25-3m^2} = \frac{10m\sqrt{3}+6m^2-5m^2\sqrt{2}-m^3\sqrt{6}}{25-3m^2}$$

$$47) \frac{2b-5\sqrt{2b}}{-1+\sqrt{2b^4}} = \frac{2b-5\sqrt{2b}}{-1+b^2\sqrt{2}} \left(\frac{-1-\sqrt{2}}{-1-\sqrt{2}} \right) = \frac{-2b-2b^3\sqrt{2}+5\sqrt{2b}-5b^2\sqrt{4b}}{1-2b^4} =$$

$$\frac{-2b-2b^3\sqrt{2}+5\sqrt{2b}-2\cdot 5b^2\sqrt{b}}{1-2b^4} = \frac{-2b-2b^3\sqrt{2}+5\sqrt{2b}-10b^2\sqrt{b}}{1-2b^4}$$

$$49) \frac{2-\sqrt{2x}}{4x-5\sqrt{3x^3}} = \frac{2-\sqrt{2x}}{4x-5x\sqrt{3x}} \left(\frac{4x+5x\sqrt{3x}}{4x+5x\sqrt{3x}} \right) = \frac{8x+10x\sqrt{3x}-4x\sqrt{2x}-5x\sqrt{6x^2}}{16x^2+25x^2(3x)} =$$

$$\frac{8x+10x\sqrt{3x}-4x\sqrt{2x}-5x^2\sqrt{6}}{16x^2+75x^2} = \frac{x(8+10\sqrt{3x}-4\sqrt{2x}-5x\sqrt{6})}{x(16x+75x)} = \frac{8+10\sqrt{3x}-4\sqrt{2x}-5x\sqrt{6}}{16x+75x}$$

$$51) \frac{-4p-\sqrt{p}}{-p-\sqrt{p^3}} = \frac{-4p-\sqrt{p}}{-p-p\sqrt{p}} \left(\frac{-p+p\sqrt{p}}{-p+p\sqrt{p}} \right) = \frac{4p^2-4p^2\sqrt{p}+p\sqrt{p}-p\sqrt{p^2}}{p^2-p^2\cdot p} = \frac{4p^2-4p^2\sqrt{p}+p\sqrt{p}-p^2}{p^2-p^3} =$$

$$\frac{3p^2-4p^2\sqrt{p}+p\sqrt{p}}{p^2-p^3} = \frac{p(3p-4p\sqrt{p}+\sqrt{p})}{p(p-p^2)} = \frac{3p-4p\sqrt{p}+\sqrt{p}}{p-p^2}$$